

Exercise 36

Evaluate the integral.

$$\int_1^{18} \sqrt{\frac{3}{z}} dz$$

Solution

Rewrite the integrand as a monomial, which can be integrated easily by using the power rule in reverse.

$$\begin{aligned} \int_1^{18} \sqrt{\frac{3}{z}} dz &= \int_1^{18} \sqrt{3} \sqrt{\frac{1}{z}} dz \\ &= \int_1^{18} \sqrt{3} z^{-1/2} dz \\ &= \sqrt{3} \int_1^{18} z^{-1/2} dz \end{aligned}$$

Apply the second part of the fundamental theorem of calculus.

$$\begin{aligned} \int_1^{18} \sqrt{\frac{3}{z}} dz &= \sqrt{3} \left(\frac{z^{1/2}}{\frac{1}{2}} \right) \Big|_1^{18} \\ &= 2\sqrt{3} (z^{1/2}) \Big|_1^{18} \\ &= 2\sqrt{3} (18^{1/2} - 1^{1/2}) \\ &= 2\sqrt{3} (3\sqrt{2} - 1) \end{aligned}$$